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TIME DOMAIN INTEGRAL EQUATION APPROACH TO EM SCATTERING BY DIELECTRIC SOLIDS†

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The responses of homogeneous, nondispersive dielectric solids of arbitrary 3-dimensional shape are computed in the time domain. Surface space-time integral equations are written for both $\vec{E}$ and $\vec{H}$, both external and internal. The four equations are solved for the surface fields induced by a smooth incident pulse, subject to the boundary conditions. For example, the integral equation (Kirchhoff) for external $\vec{H}$ on the surface is

$$\vec{H}(\vec{r},t) = 2\vec{H}^{inc}(\vec{r},t) + \qquad (1)$$

$$+ \frac{1}{2\pi} \int_S \left\{ \frac{1}{R} \hat{n}' \times \sqrt{\frac{\varepsilon}{\mu}} \frac{\partial \vec{E}'}{c\partial t} + \mathcal{L} \left((\hat{n}' \cdot \vec{H}')\hat{R} + (\hat{n}' \times \vec{H}') \times \hat{R} \right) \right\} dS'$$

where $\vec{E}'$ means $\vec{E}(\vec{r}',\tau)$, $\tau = t-R/c$, $\vec{R} = \vec{r} - \vec{r}'$, $\mathcal{L} = \frac{1}{R^2} + \frac{1}{Rc}\frac{\partial}{\partial\tau}$

These equations are solved numerically by marching in time, a technique made possible by the fact that the arguments under the integral are retarded in time. Having solved for the surface fields, the far scattered field is computed from a similar integral. A smooth Gaussian incident pulse is used with pulse width about a target dimension so that the computed response is valid from DC to well into the resonance region. The calculated results are verified by comparison with the Mie series solution for the case of a sphere and by comparison with time domain measurements for the cases of sphere capped- and right circular-cylinders.

The method is an extension of the time domain approach used for scattering from conducting target [1]. A time domain approach was used in [2] for dielectric slabs. Most other dielectric scattering work has been done in the frequency domain. An advantage of the time domain approach is that it avoids the need for matrix inversion. In addition, the time domain response can be suggestive of target geometry.

A technique similar to that described by Mautz and Harrington [3] for frequency domain solutions is used here to reduce the four

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dependent field equations to two. Namely, the external and internal equations on the surface are linearly combined by the (more or less arbitrary) constants β and α :

$$\vec{H} + \beta\vec{H}_{in} = \text{sum of H-integrals}$$

and $$\vec{E} + \alpha\vec{E}_{in} = \text{sum of E-integrals} \tag{2}$$

where β and α are here chosen to be approximately $1/\varepsilon_r$ and $1/\mu_r$ respectively.

The solution of the scattering problem is completely specified by the value of the tangential components $\vec{J} \equiv \hat{n} \times \vec{H}$ and $\vec{M} \equiv \vec{E} \times \hat{n}$ on the surface, and by the boundary conditions that these be continuous across the boundary. However, the Kirchhoff integrals also contain the normal components E_n and H_n. These could be eliminated by continuity relations, an approach which requires the numerical evaluation of surface derivatives. Here, instead, the normal components are computed directly from Eq.(1) and its three companions, and the normal boundary conditions are applied. Despite the apparent redundancy in formulation, the solution is numerically stable. Avoiding the need for space differentiation means that the space time integral equation technique can be applied to arbitrary 3-dimensional solids without further difficulty.

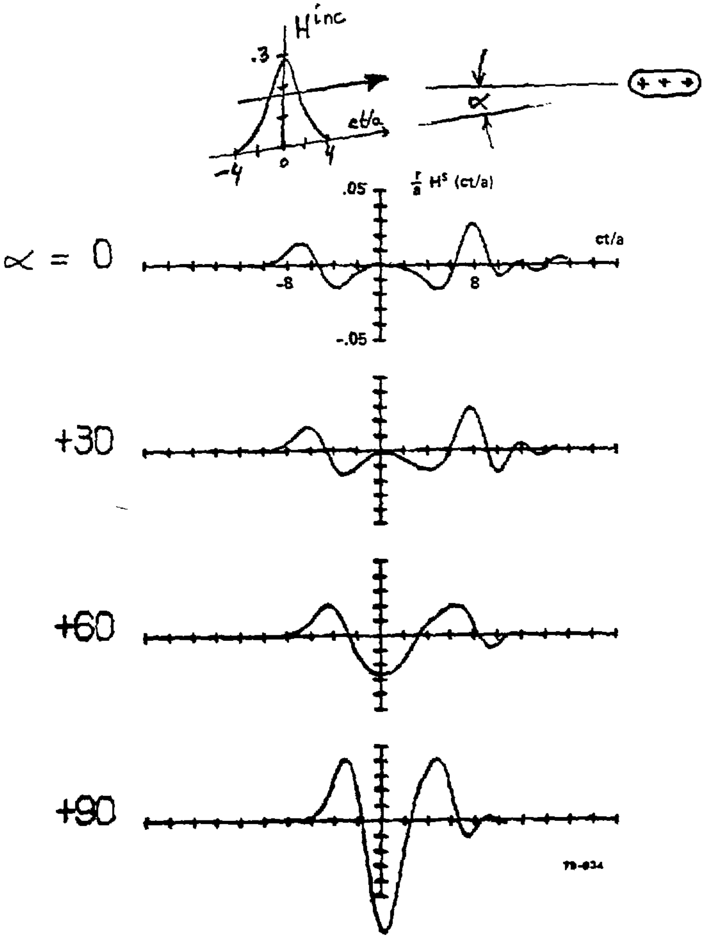
The response of a nylon sphere-capped cylinder ($\varepsilon_r = 3$) is illustrated as a function of aspect. The incident pulse, target, and response, are shown to scale with time in units of ct.

References

[1] C.L. Bennett and G.F. Ross, "Time Domain Electromagnetics and Its Applications," Proc. IEEE, March 1978, pp.299-318.

[2] J.Ch. Balomey, Ch. Durix, and D. Lesselier, "Time Domain Integral Equation Approach for Inhomogeneous and Dispersive Slab Problems," IEEE Trans. AP-26, September 1978.

[3] J.R. Mautz and R.F. Harrington, "Electromagnetic Scattering from a Homogeneous Body of Revolution," TR-77-10, Dept. of Electrical and Computer Engineering, Syracuse University, November 1977.



Smoothed impulse response — sphere capped cylinder: $a_n = 1/2$, $\epsilon_r = 3$, TE (STIE).