Errata to

Sharp Bounds on Random Walk Eigenvalues via Spectral Embedding

Int. Math. Res. Not. IMRN **2018**, no. 24 (2018), 7555–7605.

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In Theorem 1.3, the range of k ought to be $2 \le k \le n$.

Fact 3.3 ought to say that $\mu^*(\lambda_k) \ge (k-1)/n$.

Equation (3.4) is said on p. 7567 to be proved in Proposition A.1, but that proposition assumes the random walk is lazy. In fact, the assumption of laziness is not needed. To see this, replace the last sentence of its proof by the following: For any norm $\|\bullet\|$, such as ℓ^∞, and functions g_x, such as $g_x^{(t)} \colon y \mapsto \dfrac{p_t(x,y)}{\pi(y)} - 1$, we have

$$\Big\|\sum_z p(x,z) g_z\Big\| \le \sum_z p(x,z)\|g_z\| \le \max_z \|g_z\|.$$

Since $g_x^{(t+1)} = \sum_z p(x,z) g_z^{(t)}$, it follows that $\max_x \|g_x^{(t+1)}\| \le \max_x \|g_x^{(t)}\|$, which gives the result.

The proof of Proposition 4.2 does not need its second paragraph, because $\mu_x^*(\delta) > 0$ always, due to the assumption that $\delta \ge \lambda_2$.

The proof of Corollary 4.3 ought to cite the following, rather than (3.4): For a lazy random walk, we have

$$\left|\frac{p_t(x,y)}{\pi(y)} - 1\right| \le \sqrt{\frac{p_t(x,x)}{\pi(x)} - 1}\,\sqrt{\frac{p_t(y,y)}{\pi(y)} - 1},$$

whence

$$\tau_\infty(\epsilon) = \min\Big\{t : \forall x \in V \;\; \frac{p_t(x,x)}{\pi(x)} \le 1 + \epsilon\Big\}.$$

To see this, subdivide each edge by one vertex, with the two resulting edges getting the same weight as the original edge. Let $\tilde{p}_t$ denote the resulting transition probabilities. Then for all x, y in the original graph, $p_t(x,y) = \tilde{p}_{2t}(x,y)$. Apply to the new random walk all of the proof of Proposition A.1 except the last sentence to see the claimed inequality.

The definitions of $\rho_0(G)$ and $\rho(G)$ on p. 7578 should both use the inner product relative to π, i.e., $\langle \cdot, \cdot \rangle_\pi$.

In Lemma 4.5, the hypothesis that G is loopless is missing.

On p. 7583, the inequality in the last line should have 4 in place of 2: $p_t(x, x) \leq 4w(x)/\sqrt{t+1}$.

On p. 7590, at the end of the second paragraph, y should be x.

The bounds from [42] on the bottom of p. 7596 and the bottom of p. 7597 were cited slightly incorrectly by virtue of missing the dependence on the degree, d. In addition, [42] assumes that the graph is amenable. The correct bounds are

$$p_{2t}(x, x) \leq \frac{(2+D)^{1+D/2}(2d)^{D/2}}{C}(2t)^{-D/2}$$

and

$$c_5 < \frac{e^{2/a-1}(ac_2)^{2/a}}{4^{(a+1)/(a+2)}d}.$$

In the former case, our bound is better except when $d = 2$, when ours is worse by about 4%.

On p. 7602 in the first display, the exponential starts as $-\lambda/ - c_2$, which should be $-\lambda t - c_2$.

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22 March 2022