Errata to

Upper bounds for the spectral function on homogeneous spaces via volume growth

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by Chris Judge and Russell Lyons

On p. 1839, we allowed complex-valued functions on M and thus needed to complexify TM. We left g to be bilinear, not sesquilinear, which means that on p. 1840, we should have defined $|X|$ to be $\sqrt{g(X,\bar{X})}$. However, there was no need for complex-valued functions.

On p. 1846, we should have used $-\Delta_x$ in the heat equation, because we used the geometers' (and probabilists') sign convention to define Δ.

The proof of Theorem 4.9 omitted a discussion of curve length in $\mathcal{H}$ and how it relates to $\nabla_x F_x^\lambda$. Here is one way to proceed.

First, a C^1 map $\alpha\colon I \to \mathcal{H}$ for an interval $I \subseteq \mathbb{R}$ is a function with a norm-derivative α' that is norm continuous. The length of the resulting curve is defined to be $\int_I \|\alpha'(t)\|\,dt$.

Now let $\gamma\colon I \to M$ be a smooth curve on a compact interval, I. Fix $\lambda > 0$. Consider the curve $\alpha\colon t \mapsto F^\lambda_{\gamma(t)}$ in $\mathcal{H}$. Thus, $\alpha(t)\colon y \mapsto e_\lambda\big(\gamma(t), y\big)$ for $y \in M$. Define the smooth, symmetric function $\eta\colon I \times I \to \mathbb{R}$ by $\eta(s,t) := e_\lambda\big(\gamma(s),\gamma(t)\big)$. We claim that

$$\frac{\alpha(s)-\alpha(t_0)}{s-t_0} - \frac{\alpha(t)-\alpha(t_0)}{t-t_0}$$

has small norm when $|s-t_0|+|t-t_0|$ is small (uniformly in I^3). If we compute the square norm and use the fact that e_λ is the projection kernel for $\mathcal{H}_\lambda$, then we obtain the value

$$\frac{\eta(s,s)+\eta(t_0,t_0)-2\eta(s,t_0)}{(s-t_0)^2} + \frac{\eta(t,t)+\eta(t_0,t_0)-2\eta(t,t_0)}{(t-t_0)^2} \\ - 2\frac{\eta(s,t)+\eta(t_0,t_0)-\eta(s,t_0)-\eta(t,t_0)}{(s-t_0)(t-t_0)}.$$

Expanding η in a Taylor series about t_0, we see that it suffices to check that the square norm is 0 when η is replaced by a symmetric polynomial of degree at most 2. In fact, it suffices to check the cases where $\eta(u,v)$ is replaced by a constant; by $(u-t_0)+(v-t_0)$; by $(u-t_0)\cdot(v-t_0)$; or by $(u-t_0)^2+(v-t_0)^2$. Each is straightforward.

It follows that $\frac{\alpha(s)-\alpha(t_0)}{s-t_0}$ converges in norm as $s \to t_0$. Therefore, the norm limit α' equals its weak limit, which is the function $y \mapsto \gamma'(t_0)F^\lambda_y = g\big(\nabla F^\lambda_y\big(\gamma(t_0)\big), \gamma'(t_0)\big)$. Finally, to show that α' is continuous, it suffices to show that

$$\frac{\alpha(s)-\alpha(t_0)}{s-t_0} - \frac{\alpha(s)-\alpha(t_1)}{s-t_1}$$

has small norm when $|s - t_0| + |s - t_1|$ is small. But this is the same as what we first showed.

This gives that the length of α is at most the length of γ times

$$\sup_{t_0 \in I} \big\| g\big(\nabla F_{\bullet}^{\lambda}\big(\gamma(t_0)\big), \gamma'(t_0)\big) / |\gamma'(t_0)| \big\| \le \sup_{x \in M} \| |\nabla_x F_x^{\lambda}| \|$$

by the Cauchy–Schwarz inequality.

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