Erratum to

Indistinguishability of Percolation Clusters

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by Russell Lyons and Oded Schramm

In the proof of Lemma 3.6, $\Pi_e\mathcal{G} = \mathcal{G}$ should be $\Pi_e\mathcal{G} \cup \Pi_{\neg e}\mathcal{G} = \mathcal{G}$.

In the proof of Theorem 3.3, just before the first displayed equation, $n \in \mathbb{Z}$ should be $n \in \mathbb{N}$. Replace the first displayed equation by

$$Y_{n,e} := \widehat{\mathbf{P}}[\mathcal{E}_e^n \mid \omega] \text{ is } \mathcal{F}_{\neg e}\text{-measurable.}$$

Replace the second displayed equation by

$$\begin{aligned}\widehat{\mathbf{P}}[\mathcal{E}_e^n \cap \Pi_e\mathcal{B}] &= \mathbf{E}\big[\widehat{\mathbf{P}}[\mathcal{E}_e^n \cap \Pi_e\mathcal{B} \mid \mathcal{F}]\big] = \mathbf{E}\big[Y_{n,e}\mathbf{1}_{\Pi_e\mathcal{B}}\big] = \mathbf{E}\big[Y_{n,e}\,\mathbf{P}[\Pi_e\mathcal{B} \mid \mathcal{F}_{\neg e}]\big] \\ &= \mathbf{E}\big[Y_{n,e}\mathbf{1}_{\Pi_{\neg e}\mathcal{B}\cup\Pi_e\mathcal{B}}\,\mathbf{P}[e \in \omega \mid \mathcal{F}_{\neg e}]\big] = \mathbf{E}\big[Y_{n,e}\mathbf{1}_{\Pi_{\neg e}\mathcal{B}\cup\Pi_e\mathcal{B}}Z(e)\big] \\ &\geq \mathbf{E}\big[Y_{n,e}\mathbf{1}_{\mathcal{B}}Z(e)\big] \geq \delta\,\mathbf{E}\big[Y_{n,e}\mathbf{1}_{\mathcal{B}}\big] = \delta\,\widehat{\mathbf{P}}[\mathcal{E}_e^n \cap \mathcal{B}]\,.\end{aligned}$$

The claim towards the end of the proof that $\mathcal{B}_m$ is Γ-invariant is true when $W(0)$ is not fixed to be o. Furthermore, we still have $\widehat{\mathbf{P}}[\mathcal{B}_m] = \widehat{\mathbf{P}}[\mathcal{E}^m \cap \mathcal{A}_o \cap \mathcal{P}_{e_m}]$. This allows the application of Lemma 3.13, with the same conclusion. However, e_m is random and not part of the setting of Lemma 3.13. That lemma can be extended easily, or one can simply write that the probability in question is the expectation of the sum of $\mathbf{1}_{[\mathcal{E}^m\cap\mathcal{A}_{W(0)}\cap\mathcal{P}_e^m]}/s_r$ over e at distance r from $W(m)$, where $\mathcal{P}_e^m$ is the event that e is pivotal for $C\big(W(m)\big)$ and s_r is the number of edges at distance r from o.

The three sentences of the first paragraph of the proof of Lemma 4.2 beginning "Let $\gamma \in \Gamma$" should be replaced by the following: "To prove that freq is Γ-invariant, note that for every $\gamma \in \Gamma$, there is an $m \in \mathbb{N}$ such that with positive probability $X(m) = \gamma o$. Hence for every measurable $A \subset [0,1]$ such that $\alpha(C) \in A$ with positive probability, we have $\alpha(\gamma C) \in A$ with positive probability. A similar argument shows that if $\alpha(\gamma C) \in A$ with positive probability, then $\alpha(C) \in A$ with positive probability."

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