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ERRATUM TO "MEASURE-THEORETIC QUANTIFIERS AND HAAR MEASURE"

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Dr. Henry Schaerf has kindly pointed out the following gap in the application of Proposition 1 of [**1**] to the main example of a measurable predicate, "$st \notin E$", where E is a Borel subset of a topological group G. Namely, this predicate is measurable with respect to the Borel subsets $\mathcal{B}(G \times G)$ of $G \times G$, but not necessarily with respect to $\mathcal{B}(G) \times \mathcal{B}(G)$, the smallest σ-algebra containing $\{E \times F : E, F \in \mathcal{B}(G)\}$. Hence Proposition 1 does not apply. (An example where $\mathcal{B}(X \times X) \neq \mathcal{B}(X) \times \mathcal{B}(X)$ for a topological space (though, admittedly, not for a group) is given in [**2**, p. 222, (17–17)].) Since this mistake is clearly easy to make (see also [**4**]) and yet the needed form of Fubini's theorem is difficult to find, it seems worthwhile to make a careful correction.

The easiest rectifying assumption to make is that all groups in [**1**] satisfy the second axiom of countability. It is then easy to show that $\mathcal{B}(G \times G) = \mathcal{B}(G) \times \mathcal{B}(G)$ and a standard form of the Fubini-Tonelli theorem [**3**] applies.

Another way to rectify [**1**] is to assume that all spaces are locally compact Hausdorff and that all measures on them are complete and regular (as well as σ-finite). Here, we are using the following

DEFINITION [**2**, p. 109]. If X is a locally compact Hausdorff space, $\mathcal{M}$ a σ-algebra in X, and μ a positive measure on $\mathcal{M}$, then μ is called *regular* if the following conditions hold:

(i) $\mathcal{M}$ contains all open sets;

(ii) $\mu F < \infty$ if F is compact;

(iii) if G is open, $\mu G = \sup\{\mu F : F \subset G,\ F \text{ compact}\}$;

(iv) if $A \in \mathcal{M}$, $\mu A = \inf\{\mu G : A \subset G,\ G \text{ open}\}$.

A complex measure μ is *regular* if $|\mu|$ is.

We call a function f on a positive measure space $(X, \mathcal{M}, \mu)$ *μ-summable* if f is $\mathcal{M}$-measurable and $\int_X |f|\, d\mu < \infty$. From [**2**, Theorems 17.12 and 17.13 on p. 215, Theorem 17.8 on p. 212, and pp. 199–200], we have the following form of

THE FUBINI-TONELLI THEOREM. *Let X_i $(i = 1, 2)$ be locally compact Hausdorff spaces and $X = X_1 \times X_2$ with the product topology. Let μ_i be positive complete regular σ-finite measures on X_i for $i = 1, 2$. Then there exists a unique complete regular σ-finite measure ν on X such that:*

(i) *If f is a ν-summable function on X, then $\forall^e y[\mu_2]$ $f(x,y)$ is μ_1-summable as a function of x; $\int_{X_1} f(x,y)\, d\mu_1(x)$, which is defined a.e. $[\mu_2]$, is μ_2-summable; likewise with the roles of μ_1 and μ_2 reversed; and*

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$$(*)\qquad \begin{aligned}\int_X f\,d\nu &= \int_{X_2}\left(\int_{X_1} f(x,y)\,d\mu_1(x)\right)\,d\mu_2(y)\\ &= \int_{X_1}\left(\int_{X_2} f(x,y)\,d\mu_2(y)\right)\,d\mu_1(x).\end{aligned}$$

(ii) *If f is a nonnegative ν-measurable function on X, then $\forall^e y[\mu_2]$ $f(x,y)$ is μ_1-measurable as a function of x; $\int_{X_1} f(x,y)\,d\mu_1(x)$, which is defined as an extended real number a.e. $[\mu_2]$, is μ_2-measurable; likewise with the roles of μ_1 and μ_2 reversed; and $(*)$ holds in the extended reals.*

This form of the Fubini-Tonelli theorem gives a new form of Proposition 1 of [**1**] which, with the above assumption of having complete regular measures on locally compact groups, validates the other results of [**1**].

Finally, we note a typographical error in the statement of Proposition 1 of [**1**]: "$\forall^e[\mu]$" should be "$\forall^e x[\mu]$". Also, "left" should be "right" in lines 3 and 5 of p. 69 of [**1**].

References

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